## 1. PLENARY LECTURES

### Jacobi-Angelesco multiple orthogonal polynomials

**Walter Van Assche** ( KU Leuven, Belgium)
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**Abstract:** Jacobi-Angelesco multiple orthogonal polynomials are polynomials with orthogonality conditions on $r$ disjoint intervals, with a Jacobi weight function. We will take $r$ intervals which start at 0 and end in the $r$ roots of unity, thereby forming a symmetric $r$-star. We give explicit formulas and properties of the type I and type II multiple orthogonal polynomials. We also explain how type I Legendre-Angelesco polynomials appear in the theory of multiwavelets.

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### Certain Results for the Sheffer Convolution of the Gould-Hopper Polynomials

**Subuhi Khan** (Aligarh Muslim University, Aligarh, India)
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**Abstract:** The Sheffer class contains important sequences such as the Hermite, Laguerre, Bessel, Bernoulli, Poisson-Charlier polynomials *etc.* These polynomials have applications in physics and number theory. The special polynomials of two variables provided new means of analysis for the solution of large classes of partial differential equations often encountered in physical problems. The study of hybrid and mixed type special polynomials have witnessed a significant evolution during the recent years. The importance of the 2-variable forms of the special polynomials and Sheffer sequences provided motivation for this work. In this work, the Gould-Hopper and Sheffer polynomials are combined to introduce a hybrid family namely the Gould-Hopper-Sheffer polynomials family by using operational methods. The determinant form and other properties of these polynomials are established. Examples of some members belonging to this family are considered and numbers related to some mixed special polynomials are also explored.

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### Multiple Orthogonal Polynomials and Their Properties

**Galina Filipuk** (University of Warsaw, Poland)
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**Abstract:** In this talk I shall review basic definitions and properties of multiple orthogonal polynomials (MOPs). In particular, I shall explain how to obtain differential equations for MOPs and show that the roots of some of these polynomials and of their Wronskians have regular patterns in the complex plane.

The talk is based on the following papers:

References

[1] G. Filipuk, W. Van Assche and L. Zhang, Ladder operators and differential equations for multiple orthogonal polynomials (J. Phys. A.).
[2] G. Filipuk, W. Van Assche and L. Zhang, Multiple orthogonal polynomials associated with an exponential cubic weight (J. Approx. Theory).
[3] L. Zhang and G. Filipuk, On certain Wronskians of multiple orthogonal polynomials (SIGMA).


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## Orthogonal polynomials and quantum mechanics


**Luigi Accardi** ( Rome university, Italy)
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**Abstract:** The starting point of the quantum probabilistic approach to the theory of orthogonal polynomials (OP) is an operator interpretation of the tri-diagonal Jacobi relation in terms of Creation, Annihilation and Preservation (CAP) operators. This allows to associate, in a canonical way, to any random variable with all moments commutation relations that generalize the Heisenberg commutation relations (corresponding to the Gauss-Poisson class) or anti- commutation relations (corresponding to the Bernoulli class). Since in quantum theory the Gauss and Poisson measures correspond to linear interactions, this means that other classes of measures will correspond to more complex, non-linear, interactions. From the mathematical point of view this approach has led to some new results in the theory of OP. In the talk these developments will be briefly outlined.


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## On $(m,p)$-expansive and $(m,p)$-contractive tuples of commuting operators on Banach and Hilbert spaces


**Ould Ahmed Mahmoud Sid Ahmed** (Aljouf University, Aljouf. Saudi Arabia)
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**Abstract:** The study of tuples of commuting operators was the subject of intensive study by many authors. Our aim in this work is to consider a generalization of the notions of $(m,p)$-expansive and $(m,p)$-contractive of a single operator done in $[3,6,7]$ and $[1,5]$ to the multivariable operators. We study some of the basic properties of these tuples of commuting operators.


For a $d$-tuple of commuting operators $\mathbf{T} := (T_1, \cdots, T_d) \in \mathcal{B}(\mathcal{X})^d$, we denote

$$\Lambda_l^{(p)}(\mathbf{T};\ x) := \sum_{0\le k\le l} (-1)^k \binom{l}{k} \Big( \sum_{|\beta|=k} \frac{k!}{\beta!} \|\mathbf{T}^\beta x\|_{\mathcal{X}}^p \Big),\ x \in \mathcal{X}$$

where $l \in \mathbb{N}_0,\ p \in (0,\infty)$ and $\binom{l}{k}$ denotes the binomial coefficient. We say that $\mathbf{T}$ is an $(m,p)$-expansive tuple of operators if $\Lambda_m^{(p)}(\mathbf{T};\ x) \le 0\ \ \forall\ x \in \mathcal{X}$. When such a relation is valid for $k \in \{1, \cdots, m\}$, we say that $\mathbf{T}$ is an $(m,p)$-hyperexpansive tuple. Moreover if $\Lambda_m^{(p)}(\mathbf{T};\ x) \ge 0\ \ \forall\ x \in \mathcal{X}$, we say that $\mathbf{T}$ is an $(m,p)$-contractive tuple and if $\mathbf{T}$ is $(k,p)$-contractive tuple for all positive integer $k \le m$, the map $\mathbf{T}$ is said $(m,p)$- hypercontractive tuple. These concepts extend the definitions of $m$-isometries and $(m,p)$-isometries tuples of bounded linear operators acting on Hilbert or Banach spaces was introduced and studied in [2] and [4].

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## Dynamics of disjoint sequences of operators

**Mohamed Amouch** (University Chouaib Doukkali, Eljadida, Morocco)
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**Abstract:** A sequence $(T_n)_{n\geq 0}$ of operators on a Banach space $X$ is called hypercyclic or universal provided it supports some vector $x$ in $X$ whose orbit $\{T_n(x),\ n \geq 0\}$ is dense in $X$. Such $x$ is called a universal vector for the family $(T_n)_{n\geq 0}$. A $N \geq 2$ sequence

$$(T_{1,j})_{j=1}^{\infty}, (T_{2,j})_{j=1}^{\infty}, ..., (T_{N,j})_{j=1}^{\infty}$$

of operators on $X$ is said to be disjoint or diagonally subspace universal respect to a nonzero subspace $M$ of $X$ provided some vector $(x, x, ..., x)$ in the diagonal of $X^N$, such that $\{(T_{1,j}x, T_{2,j}x, ..., T_{N,j}x, ), j \in \mathbb{N}\} \cap M^N$ is dense in $M^N$. Such vector $x$ is called a $d - M$ universal vector.
We contrast the standard dynamical properties of a single operator and a sequence of operator versus the disjoint dynamics of $N \geq 2$ sequences

$$(T_{1,j})_{j=1}^{\infty}, (T_{2,j})_{j=1}^{\infty}, ..., (T_{N,j})_{j=1}^{\infty}$$

of operators on $X$.

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## The Problem of invariant and Hyperinvariant subspaces An affirmative answer to the invariant subspace problem for hyponormal operators

**Salah Mecheri** (Tebessa University, College of Science, Algeria)
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**Abstract:** One of the most difficult unsolved problems in functional analysis is the invariant subspace problem. This is the question: does every operator on an infinite-dimensional Hilbert space $H$ have a non-trivial closed invariant subspace? This problem is open for more than half a century. A subnormal operator has a non-trivial closed invariant subspace, but the existence of non-trivial closed invariant subspace for a hyponormal operator $T$ still open. In this paper we give an affirmative answer of the existence of a non-trivial closed invariant subspace for a hyponormal operator in a Hilbert space. More generally, we show that a large classes of operators containing the class of hyponormal operators have non-trivial invariant subspaces. Finally, we show that every generalized scalar operator on a Banach space has a non-trivial closed invariant subspace.

## 2. SHORT COMMUNICATIONS

### Cubic decomposition of a family of semiclassical orthogonal polynomials of class two

**Amel Saka** (Gabes University, Tunisia)
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**Abstract:** We deal with a family of semiclassical orthogonal polynomial sequences of class two having the cubic decomposition $W_{3n}(x) = P_n(x^3), n \geq 0$. Only four monic orthogonal polynomial sequences (MOPS) appear which their recurrence coefficients are explicitly given.

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### Orthogonal polynomials associated with an inverse spectral transform. The cubic case

**Lamaa Khaled** (Gabes University, Tunisia)
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**Abstract:** The purpose of this Talk is to give some new algebraic properties of the orthogonality of a monic polynomial sequence $\{Q_n\}_{n\geq 0}$ defined by

$$Q_n(x) = P_n(x) + s_n P_{n-1}(x) + t_n P_{n-2}(x) + r_n P_{n-3}(x), \quad n \geq 1,$$

where $r_n \neq 0, n \geq 3$, and $\{P_n\}_{n\geq 0}$ is a given sequence of monic orthogonal polynomials. Essentially, we consider some cases in which the parameters $r_n$, $s_n$, and $t_n$ can be computed more easily. Also, as a consequence, a matrix interpretation using $LU$ and $UL$ factorization is done. Some applications for Laguerre, Bessel and Tchebychev orthogonal polynomials of second kind are obtained.

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### Calderón's Reproducing Formula and Uncertainty Principle for the Continuous Wavelet Transform associated with the $q$-Bessel Operator

**Bochra Nefzi**
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**Abstract:** In this paper, we present some new elements of harmonic analysis related to the $q$-Bessel Fourier transform introduced earlier in [1, 2], we define and study the $q$-wavelet and the continuous $q$-wavelet transform associated with the $q$-Bessel operator. Thus, some results (Plancherel's formula, inversion formula, etc.) are established. Next, we prove a Calderón's formula and an analogue of Heisenberg's inequality for the continuous $q$-wavelet transform.

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# Boundeed operators in Dunkl setting
# $L^p$ estimates for an oscillating Dunkl multiplier

**Gaidi Mohamed** (Faculty of Sciences of Tunis)
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**Abstract:** We study the $L^p$ boundedness of a class of oscillating multiplier operator for the Dunkl transform , $T_{m_\alpha} = \mathcal{F}_k^{-1}\big(m_\alpha \mathcal{F}_k(f)\big)$ with $m(\xi) = |\xi|^{-\alpha} e^{i|\xi|}\phi(\xi)$ . We obtain an $L^p$ - bound result for the corresponding maximal functions . As a specific applications , we give an extension of the $L^p$ estimate for the wave equation and of Stein's theorem for the analytic family of maximal spherical means .

Let $\alpha < 0$ and $\beta < 0$ , the oscillating multiplier $T_{\alpha,\beta}$ is defined via Fourier transform by $\mathcal{F}\big(T_{\alpha,\beta}\big) = m_{\alpha,\beta}\mathcal{F}(f)$ , where $m_{\alpha,\beta} = |\xi|^{-\alpha} e^{i|\xi|^\beta}\phi(\xi)$ and $\phi$ is a $\mathcal{C}^\infty$ function on $\mathbb{R}^n$ which vanishes near the origin and is equal to 1 for all sufficiently large $\xi$ . The study of their $L^p$ properties going back to the works of I.Hirschman [7] in the case $n = 1$ and S.Wainger [15] in higher dimensions . Later on , they have been extensively studied again by many authors in several different contexts , see $[6, 8, 9, 11]$ . This paper is devoted to the study of $L^p$ boundedness of oscillating multiplier in the context of Dunkl analysis . We will focus on the case $\beta = 1$ , because of the close connection to the wave equation associated with the Dunkl Laplacian $\Delta_k$ , and to the spherical maximal function . The latter is already studied by L.Deleaval [4] . In order to describe more precisely the results studied in the paper , we shall start by giving a brief summary of the Dunkl analysis.

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## Wavelets and generalized windowed transforms associated with the Dunkl-Bessel Laplace operator on $\mathbb{R}^d \times \mathbb{R}_+$

**Amina Hassini**
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**Abstract:** In this presentation we study Wavelets and the generalized windowed transform associated with the Dunkl-Bessel Laplace operator on $\mathbb{R}^d \times \mathbb{R}_+$ and we prove for this transform Plancherel and inversion formulas.

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## Numerical solution of nonlinear stochastic integral equations using stochastic operational matrix based on Bernoulli polynomials

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**Abstract:** In this paper, a computational numerical method for solving nonlinear stochastic differential integral equation is presented. The method is based on Bernoulli polynomials approximation and their stochastic operational matrix of integration and the so called collocation method. The stochastic integral equations are reduced to systems of algebraic equations with unknown Bernoulli coefficients. Numerical examples illustrates the efficiency and applicability of the technique.

**Key-Words** Stochastic integral equations, Bernoulli polynomials, stochastic operational matrix of integration, Collocation method.

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## Spectral properties of $(A, m)$-isometric Operator Tuples on Semi-Hilbertian Spaces

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**Abstract:** A $d$-tuple of commuting bounded linear operators $\mathbf{T} = (T_1, T_2, \ldots, T_d)$ acting on a Hilbert space $\mathbb{H}$ is called an $(A, m)$-isometry if

$$\sum_{j=0}^{m} (-1)^j \binom{m}{j} \sum_{|\alpha|=j} \frac{j!}{\alpha!} \mathbf{T}^{*\alpha} A \mathbf{T}^{\alpha} = 0,$$

where $A$ is a positive operator ($A \neq 0$) and $\alpha = (\alpha_1, \ldots, \alpha_d) \in \mathbb{N}^d$. The aim of the present paper is to study the $(A, m)$-isometries on a semi-Hilbertian space. We give basic properties and we establish some spectral results that generalizes those given in [GR] and [SAS].

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## The structure of $A$-$m$-isometric weighted shift operators

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**Abstract:** In this paper, we will study the characterization of $A$-$m$-isometric unilateral and bilateral weighted shift operators. We shall prove that any such operator is a $A$-Hadamard product of $A$-2-isometries and $A$-3-isometries. We also study weighted shift operators whose powers are $A$-$m$-isometric.

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## (A,m)-isometries on Hilbert spaces

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**Abstract:** A bounded linear operator $T$ on a Hilbert space $H$ is called an $(A,m)$-isometry, for some positive operator $A$ on $H$ and integer $m$ if

$$\sum_{k=0}^{m}(-1)^{m-k}\binom{m}{k}T^{*k}AT^{k}=0.$$

We give some properties of $(A,m)$-isometries. In particular, we focus on the spectral picture and the relation between $(A,m')$-isometries and $m$-isometries. Moreover, we prove that the perturbation of $(A,m)$-isometry by a bigger class than nilpotent operators is not $N$-supercyclic.

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## Critres d'explosion pour les équations de Navier-Stokes dans les espaces de Sobolev

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**Abstract:** On considre le systme Navier-Stokes incompressible en 3D:

$$(NS)\qquad \begin{cases} \partial_t v - \nu\Delta v + (v.\nabla)v = -\nabla p, & \text{dans } \mathbb{R}^+\times\mathbb{R}^3,\\ \operatorname{div} v = 0 \quad \text{dans } \mathbb{R}^+\times\mathbb{R}^3, \\ v(0) = v^0 \quad \text{dans } \mathbb{R}^3, \end{cases}$$

où $v$ dsigne la vitesse de fluide, $p$ est la pression de fluide et $\nu$ est un réel strictement positif qui désigne la viscosité. On rappelle aussi que si la condition initial est régulier, on peut exprimer $p$ en fonction de $v$. On montre qu'on a existence et unicité dans les espaces $\dot{H}^s(\mathbb{R}^3)$ (pour $1/2 < s < 3/2$) et $H^s(\mathbb{R}^3)$ (pour $s \geq 3/2$). On montre aussi qu'on dispose d'un critère d'explosion si $v \in C([0,T^*[,\dot{H}^s)$ avec $T^* < \infty$.

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## Critères d'explosion pour les équations de Navier-Stokes dans les espaces de Fourier

**Mariem Ben Naceur** (Faculty of Sciences Gabes 6072, Tunisia)
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**Abstract:** Il est bien que les équations de Navier-Stokes incompressibles restent le modèle de la mécanique de fluides le plus étudiés par les mathématiciens. Le système Navier-Stokes incompressible en trois dimensions est défini par

$$(NS)\qquad \begin{cases} \partial_t v - \nu\Delta v + (v.\nabla)v = -\nabla p, & \text{dans } \mathbb{R}^+\times\mathbb{R}^3,\\ \operatorname{div} v = 0 \quad \text{dans } \mathbb{R}^+\times\mathbb{R}^3, \\ v(0) = v^0 \quad \text{dans } \mathbb{R}^3, \end{cases}$$

où $v$ désigne la vitesse de fluide, $p$ est la pression de fluide et $\nu$ est un réel strictement positif qui désigne la viscosité. On rappelle aussi que si la condition initial est régulier, on peut exprimer $p$ en fonction de $v$. On étudie alors le système $(NS)$ dans l'espace critique $\mathcal{X}^{-1}(\mathbb{R}^3)$ avec $\|f\|_{\mathcal{X}^{-1}} = \int \frac{|\widehat{f}(\xi)|}{|\xi|}d\xi$. On montre que, si $v^0 \in \mathcal{X}^{-1}(\mathbb{R}^3)$, alors il existe une unique solution dans $C([0,T],\mathcal{X}^{-1}(\mathbb{R}^3))$. On montre aussi que toute solution est dans l'espace $L^2([0,T],\mathcal{X}^1(\mathbb{R}^3))$ avec un critère d'explosion dans le cas d'une solution

non régulière.

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## Etude asymptotique des équations de Navier-Stokes dans les espaces de Fourier

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**Abstract:** Le système Navier-Stokes incompressible en trois dimensions est défini par

$$(NS) \qquad \begin{cases} \partial_t v - \nu\Delta v + (v.\nabla)v = -\nabla p, \quad \text{dans } \mathbb{R}^+ \times \mathbb{R}^3, \\ \operatorname{div} v = 0 \quad \text{dans } \mathbb{R}^+ \times \mathbb{R}^3, \\ v(0) = v^0 \quad \text{dans } \mathbb{R}^3, \end{cases}$$

où $v$ désigne la vitesse de fluide, $p$ est la pression de fluide et $\nu$ est un réel strictement positif qui désigne la viscosité. On rappelle aussi que si la condition initial est régulier, on peut exprimer $p$ en fonction de $v$. On étudie alors le système $(NS)$ dans l'espace critique $\mathcal{X}^{-1}(\mathbb{R}^3)$ avec $\|f\|_{\mathcal{X}^{-1}} = \int \frac{|\widehat{f}(\xi)|}{|\xi|} d\xi$. On montre qu'on existence et unicité de solution dans l'espace $C([0,T],\mathcal{X}^{-1}(\mathbb{R}^3))$ avec existence globale si $\|v^0\|_{\mathcal{X}^{-1}} < \nu$. Enfin, on montre que $\lim_{t\to\infty} \|v(t)\|_{\mathcal{X}^{-1}} = 0$ si $v \in C(\mathbb{R}^+,\mathcal{X}^{-1}(\mathbb{R}^3))$ ce qui permet de déduire la stabilité de solutions globales.

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## Orthonormal sequences and time frequency localization Shapiro's uncertainty principles related to the Riemann-Liouville transform

**Aymen Hammami** (Faculty of Sciences of Tunis. Tunisia)
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**Abstract:** For every real number $p > 0$ , we define the $p$-dispersion $\rho_{p,\nu_\alpha}(f)$ of a measurable function $f$ on $[0,+\infty[\times\mathbb{R}$ , where $\nu_\alpha$ is some positive measure. We prove that for every orthonormal basis $(\varphi_{m,n})_{(m,n)\in\ \mathbb{N}^2}$ of $L^2(d\nu_\alpha)$ ,
the sequences $\Big(\rho_{p,\nu_\alpha}(\varphi_{m,n})\Big)_{(m,n)\in\ \mathbb{N}^2}$ , $\Big(\rho_{p,\nu_\alpha}\big(\widetilde{\mathfrak{F}}_\alpha(\varphi_{m,n})\big)\Big)_{(m,n)\in\ \mathbb{N}^2}$ can not be simultaneously bounded , where $\widetilde{\mathfrak{F}}_\alpha$ is some Fourier transform . The main tool is a time frequency localization inequality for orthonormal sequences in $L^2(d\nu_\alpha)$. On the other hand , we construct an orthonormal sequence $(\psi_{m,n})_{(m,n)\in\ \mathbb{N}^2} \subset L^2(d\nu_\alpha)$ such that the sequence $\Big(\rho_{p,\nu_\alpha}(\psi_{m,n})\rho_{p,\nu_\alpha}\big(\widetilde{\mathfrak{F}}_\alpha(\psi_{m,n})\big)\Big)_{(m,n)\in\ \mathbb{N}^2}$ is bounded .

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